Unbiased Estimation of Gross Errors in Process Measurements

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A new approach to gross error detection provides unbiased estimates and $100(1-\alpha)\%$ simultaneous confidence intervals of process variables when biased process measurements and process leaks exist. Presented in this article are estimation equations for process variables, as well as equations that help identify biased measurements and process leaks. These equations include the power function for a global test, and two types of α-level component tests and their power functions. Important strengths and weaknesses of this approach are compared to those of the serial compensation strategy, in particular, by varying the significance level (α), the variance-covariance matrix (Σ), the size of measurement bias (δ), the number of biased variables, and the sample size (N). Accuracy of δ estimation and performance in detecting the presence of process leaks (γ) are also evaluated and compared. The proposed approach has unique features that can provide a basis for improving the reconciliation of variables in process operations.

Introduction

In chemical processes, gross measurement errors can contribute to poor operations, incorrect material accounting, and inadequate process diagnosis. Gross measurement errors have a high probability of occurring if large measurement biases (δs) or large measurement variances (σs) exist. In this article, we assume that σs are at an acceptable level, only significantly large δs are possible, and the processes are at steady state. We also assume that only three possibilities exist for not satisfying material and energy balances: measurement biases (nonzero values for the δs), process leaks (nonzero values for γs), and random measurement variability (ϵs). Hence, we also identify and estimate leaks.

Researchers have commonly called both measurement biases (δs) and process leaks (γs) gross errors. Based on the statistical definition of a gross error, however, it is improper to call δ or γ a gross error. A gross measurement error can occur *because of* measurement bias (a nonzero value of δ), but δ is not a gross error. At the same time, there is an important difference between δ and γ that δ is directly associated with a measured variable (δ is its bias) and γ is not. Thus, while δ influences measurement error directly, γ does not.

Both γ and δ, however, can contribute to gross estimation errors. It is probably this connection that has caused them to be called "gross errors." The presence of either γ or δ can contribute to gross estimation errors because estimates of process variables can depend on both δ and γ. To avoid confusion, however, we will not refer to δ and γ as "gross errors," but simply as δ or γ generally and use θ occasionally when referring to *either* δ or γ.

Statistical methods have been used in chemical engineering gross error detection (GED) research for about 20 years (Narasimhan and Mah, 1987). Currently, the most widely used statistical test for identifying significant values of θ appears to be the measurement test (Mah and Tamhane, 1982). The measurement test (MT), however, can have very high type I error levels and low power (Heenan and Serth, 1986). Note that in this setting, a type I error is a *false* conclusion that $\theta \neq 0$. Similarly, power is the probability of *correctly* concluding that $\theta \neq 0$. The probability of a type II error (incorrectly concluding that $\theta = 0$) is 1 minus the power.

Narasimhan and Mah (1987) addressed high type I error levels of the MT by introducing a serial compensation strategy (SCS). This strategy attempts to identify all nonzero θs from the largest to the smallest. When the SCS makes a conclusion that $\theta_i \neq 0$, it uses an estimate for θ_i to "adjust" the appropriate process variable before searching for the next largest $\theta \neq 0$. A process variable is adjusted only once during this identifica-

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tion—compensation procedure. One drawback with this strategy is that identification can still be subject to large type I errors and low power. Furthermore, SCS-adjusted variables can have even larger errors than original observations because of large inaccuracies (due to large estimation errors) in the estimates used to adjust the original observations.

When all the θs are zero, the constrained maximum likelihood estimators (MLE) for measured process variables (μs) have three desirable properties: they are unbiased, they satisfy the physical constraints, and they have known distributions. In this setting, when an estimator is biased, its bias depends on the biases of other process variables. Thus, when multiple process measurements are biased, unbiased estimators are likely to be more accurate than biased estimators. (The adjustment of measured values to satisfy physical constraints, such as material balances, is called data reconciliation. Important uses for reconciled values include inputs to material accounting and optimization programs.) Another advantage of unbiased estimators is that they have known distributions. Therefore, it is possible to construct confidence intervals from unbiased estimators and thus, make even better inferences about known values. See Rollins (1990) for additional details on the statistical properties of the estimators when all the θs are zero.

Statistical equations presented here to control identification errors and estimation accuracy are applicable when multiple θs are nonzero and can have the same properties as the equations when all the θs are zero. That is, the proposed estimators can be unbiased, satisfy the constraint equations, and have known distributions. Because of the emphasis on unbiased estimators, we call this approach the unbiased estimation technique (UBET).

These statistical equations for identification (test statistics and their power functions) can be the basis for a variety of identification strategies. The purpose of this article is not to limit them to any particular identification strategy, rather, it is to present the equations, and discuss their development and applicability. Nevertheless, we do present an identification study to illustrate certain properties of these equations and their potential effectiveness. We are in the process of developing identification strategies using these equations, the results of which will be published later.

First, we review the mathematical model that takes into account biased measurements and process leaks, and then discuss the UBET. We next introduce the statistical details and show how unbiased, reconciled estimators for the μs with known distributions can be determined. Finally, we present a detailed simulation study to show some of the strengths and weaknesses of the proposed technique and compare it to the serial compensation (SC) technique of Narasimhan and Mah (1987).

Measurement Model with δ and γ

The measurement model that takes into account measurement biases and process leaks is as follows:

$$y = \mu + \epsilon \tag{1}$$

such that

$$A\mu = M\gamma \tag{2}$$

where

$$\epsilon \sim N_p(\delta, \Sigma) \tag{3}$$

$$M = [m_1, \cdots, m_q] \tag{4}$$

$$E[y] = \mu + \delta. \tag{5}$$

where

- y = $p \times 1$ random vector of measurements
- μ = $p \times 1$ vector of unknown true values
- ϵ = $p \times 1$ vector of random errors assumed to be multivariate normally distributed with mean, δ, and known variance-covariance matrix, Σ
- A = $q \times p$ constraint matrix, with rank $(A) = q$
- p = number of measured variables
- q = number of constraint equations
- m_j = known $q \times 1$ vector with zeros in every position but jth (Narasimhan and Mah, 1987)
- γ = unknown $q \times 1$ vector of leak constants
- δ = unknown $p \times 1$ vector of measurement biases
- E = expected value

Note that, for simplicity, the sample size is equal to 1. When the sample size is greater than 1, y is a vector of measurement means, and Σ is divided by the sample size. In addition, Σ is taken as known—an assumption which is not restrictive. (In a forthcoming article, we extend this approach for an unknown Σ.) The results of this research are restricted to the conditions of this model: they are restricted to normally distributed errors, steady state, and linear constraints.

Unbiased Estimation Technique

In this section, we discuss α-level test statistics, power functions, unbiased estimators, and confidence intervals for achieving the objectives of gross error detection to identify accurately biased measurements and process leaks and to estimate accurately process variables (μ_is).

Transformed measurement model

The UBET is developed from the following transformation of Eq. 1:

$$r = Ay = A\mu + A\epsilon \tag{6}$$

where it is assumed that $\mu_r = E[r] = \mathbf{0}$, if, and only if (iff), $\delta = \mathbf{0}$ and $\gamma = \mathbf{0}$. This assumption is necessary for the hypotheses, H_o: $\mu_r = \mathbf{0}$ vs. H_a: $\mu_r = A\delta + M\gamma \neq \mathbf{0}$, to be equivalent to H_o: $\delta = \mathbf{0}$ and $\gamma = \mathbf{0}$ vs. H_a: $\delta \neq \mathbf{0}$ or $\gamma \neq \mathbf{0}$. Although it is not possible for $M\gamma$ to equal $\mathbf{0}$ unless $\gamma = \mathbf{0}$ (M is diagonal), it is possible for $A\delta$ to equal $\mathbf{0}$ when $\delta \neq \mathbf{0}$.

When $A\delta = \mathbf{0}$ and $\delta \neq \mathbf{0}$, this situation has been referred to as "error cancellation" in literature. Error cancellation is a concern because $A\delta$ can be small when δ is large and thus, not cause rejection of H_o. The chance of error cancellation can be physically minimized if steps are taken to eliminate correlation among measurements. One way that plants can do this is by installing sensors that fail (mechanically or electronically) independent of other sensors. Other ways include eliminating common sources of calibration errors, sampling errors, and human measuring errors.

Computationally, the effect of error cancellation can be

minimized if adequate power is maintained. This is an important justification for power functions. In situations where $A\delta$ is likely to be small, the power function can be useful in specifying the needed sample size or Σ to maintain adequate power.

Global test

A global test (GT) is an analysis for the presence of bias in any measured variable or a leak at any node. (A node represents the interconnection of multiple process streams.) See Rollins (1990) for details on the importance of a GT. The GT that we recommend is the same test that previously appeared (see Tamhane and Mah, 1985). The test statistic is very well known in statistical literature and appeared there long before it appeared in chemical engineering literature. We present the GT here for two reasons: 1. it is connected with Eq. 6, the transformed measurement model; 2. to our knowledge, its power function has not previously appeared in chemical engineering literature.

We begin the development of the GT by stating the null and alternative hypotheses as follows: H_o: $\mu_r = \mathbf{0}$ vs. H_a: $\mu_r \neq \mathbf{0}$. Note that H_a is an alternative for *all* $\mu_r \neq \mathbf{0}$. Therefore, not only can it detect δ or $\gamma \neq \mathbf{0}$, but any cause of $\mu_r \neq \mathbf{0}$—nonsteady state, for example. Furthermore, r is distributed $N_q(\mu_r, \Sigma_r)$. Thus, an appropriate α level test (see Mardia et al., 1979) is to reject H_o in favor of H_a, iff

$$r^T\Sigma_r^{-1}r \geq \chi^2_{q,\alpha} \tag{7}$$

where Σ_r is assumed to be known and $\chi^2_{q,\alpha}$ is the upper (100α)th percentile of the χ^2_q distribution. The power function of the test statistic given by Eq. 7 is:

$$\begin{aligned}\beta &= \mathcal{P}[r^T\Sigma_r^{-1}r \geq \chi^2_{q,\alpha} \mid \mu_r] \\ &= \mathcal{P}[\text{noncentral } \chi^2_q \geq \chi^2_{q,\alpha} \mid \Delta^2]\end{aligned} \tag{8}$$

with

$$\Delta^2 = \mu_r^T\Sigma_r^{-1}\mu_r \tag{9}$$

where Δ^2 is called the noncentrality parameter. In Eq. 9, the sample size, N, is assumed to be one. When $N>1$, Δ^2 is proportional to N. Thus, for N fixed, the power depends only on Δ^2, which depends on μ_r. Therefore, one could study the effect of δ or γ on type II error $(1-\beta)$ by varying μ_r. See the books by Scheffé (1959) and Bickel and Doksum (1977) for discussions on noncentral probability distributions.

Unbiased estimators for δ and γ

If $\ell_i^T\mu_r = \delta_i$, then $\ell_i^T r$ is an unbiased estimator of δ_i, where ℓ is some vector of numbers. Therefore, to obtain unbiased estimators, $\ell^T\mu_r$ must be related to specific δs and γs. Consequently, the purpose of this section is to *show* how $\ell^T\mu_r$ can be related to δs and γs. Hence, we *show* how unbiased estimators, of the form $\ell^T\mu_r$, may be related to δs and γs. Note that it is not the purpose of this article to propose a strategy to *find* these ℓs. However, in the next section, we do comment on the necessary conditions for their existence and on *ways* to find these ℓs.

The derivation to show that $\ell^T\mu_r$ can be related to specific δs and γs is begun by first partitioning A, M, δ, and γ as follows. [It may be necessary to rearrange the columns of A, M, δ and γ (see Rollins, 1990, for more details).]

$$A = \begin{bmatrix} A_{11}^{uxu} & A_{12}^{ux(p-u)} \\ A_{21}^{vxu} & A_{22}^{vx(p-u)} \end{bmatrix} \tag{10}$$

$$M = \begin{bmatrix} M_{11}^{uxu} & \mathbf{0}^{uxv} \\ \mathbf{0}^{vxu} & M_{22}^{vxv} \end{bmatrix} \tag{11}$$

$$\delta = \begin{bmatrix} \delta_1^{ux1} \\ \delta_2^{(p-u)x1} \end{bmatrix} \tag{12}$$

$$\gamma = \begin{bmatrix} \gamma_1^{ux1} \\ \gamma_2^{vx1} \end{bmatrix} \tag{13}$$

such that the rank A_{11} is u. M_{22} is of rank v, where $u+v=q$. Now rewriting μ_r using Eqs. 10 to 13 gives:

$$\begin{aligned}\mu_r &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} + \begin{bmatrix} M_{11} & \mathbf{0} \\ \mathbf{0} & M_{22} \end{bmatrix}\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \\ &= \begin{bmatrix} A_{11} & \mathbf{0} \\ A_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \delta_1 \\ \gamma_2 \end{bmatrix} + \begin{bmatrix} A_{12} & M_{11} \\ A_{22} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \delta_2 \\ \gamma_1 \end{bmatrix} \\ &= B_1\theta_1 + B_2\theta_2\end{aligned} \tag{14}$$

where $\theta_{1i} = \delta_i$, $i = 1, \ldots, u$; $\theta_{1i} = \gamma_{i-u}$, $i = u+1, \ldots, q$; $\theta_{2j} = \delta_{u+j}$, $j = 1, \ldots, p-u$; $\theta_{2j} = \gamma_{j-p+q}$, $j = p-u+1, \ldots, p$. By letting

$$\ell_i^T = e_i^T B_1^{-1}, \tag{15}$$

$$\ell_i^T\mu_r = e_i^T B_1^{-1}\mu_r. \tag{16}$$

Now for $\theta_2 = \mathbf{0}$ (which also means that δ_2 and $\gamma_1 = \mathbf{0}$), then

$$\ell_i^T\mu_r = e_i^T B_1^{-1}B_1\theta_1 = \theta_i. \tag{17}$$

Thus, if $\theta_2 = \mathbf{0}$, then $\ell_i^T r$ $(i = 1, \ldots, q)$ are unbiased estimators of the components of δ and γ contained in θ_1.

Use of Eq. 15

In the derivation of Eq. 15 two important assumptions were made. The first was that the rank of B_1 was equal to q. The other was that at least p θs were zero. First, to obtain unbiased estimates using Eq. 15 both of these assumptions must hold simultaneously. That is, even if there are q or less nonzero θs, their locations may make it impossible to obtain a matrix B_1 with rank equal to q. If so, it will not be possible to obtain estimates for each of the q θs in θ_1.

Secondly, the requirement that $\theta_2 = \mathbf{0}$ is equivalent to the requirement that at most q of the θs are nonzero. This requirement seems reasonable for most industrial processes since a very large number of nonzero θs would likely mean very poor process control. Nevertheless, we have not seen a case in literature where this requirement was not met. Note that when

the number of nonzero θs is much less than q, the number of possibilities for θ_2 can be very large.

Thirdly, if $\theta_2 \neq \mathbf{0}$, Eq. 15 can produce biased estimates. That is, if θ_2 is not $\mathbf{0}$, it may have the following dependence:

$$\ell_i^T \mu_r = \theta_i + \boldsymbol{e}_i^T \boldsymbol{B}_1^{-1} \boldsymbol{B}_2 \theta_2. \tag{18}$$

Hence, by Eq. 18, $\ell_i^T \mu_r$ could be equal to a component of θ_1 plus a function of the components of θ_2 that are not zero.

Based on these three comments, we see estimation as the primary application for Eq. 15 *after* some identification procedure has identified the required θ_2. An important advantage of this approach is that less is demanded from the identification procedure. That is, rather than requiring identification for all the θs, the procedure just needs to find a proper $\theta_2 = \mathbf{0}$. *After* finding this θ_2, Eq. 15 can be used to estimate θ_1. Additionally, each estimate can be statistically tested for significance using tests that we will now give.

Test statistics

This section gives test statistics for testing general hypotheses of the form, H_o: $\ell^T \mu_r = 0$ against H_a: $\ell^T \mu_r \neq 0$. They can be used in an identification procedure to locate θs that are zero to meet the requirements of Eq. 17 for obtaining unbiased estimates. In addition, these tests may also be used to determine if estimates for θs are statistically significant.

In the general case, the distribution of $\ell^T \boldsymbol{r}$ is $N(\ell^T \mu_r, \ell^T \boldsymbol{\Sigma}_r \ell)$ for all ℓ. As described in Mardia et al. (1979), if μ_r is $\mathbf{0}$, then $\ell^T \mu_r$ is 0 for all ℓ. Furthermore, it can be shown (see Mardia et al., 1979) through the use of the union intersection principle that

$$\mathcal{P}[\ell^T \mu_r \in (\ell^T \mathbf{r} \pm b) \text{ for all } \ell] = 1 - \alpha \tag{19}$$

where

$$b = \sqrt{\chi^2_{q,\alpha} \ell^T \boldsymbol{\Sigma}_r \ell}. \tag{20}$$

Thus,

$$(\ell^T \boldsymbol{r} - b,\ \ell^T \boldsymbol{r} + b) \tag{21}$$

are simultaneous $100(1-\alpha)\%$ confidence intervals for all $\ell^T \mu_r$. They are called simultaneous confidence intervals because the probability is at least $(1-\alpha)$ that *all* confidence intervals for $\ell^T \mu_r$ are simultaneously true. Hence, Eq. 21 may be used to study specific hypotheses concerning μ_r. For example, if H_o: $\mu_r = \mathbf{0}$ is rejected at level α, then there is at least one vector ℓ for which the corresponding confidence interval does not contain zero. Thus, Eq. 21 can be used to find which ℓs led to the rejection of H_o. Note that the corresponding test to Eq. 21 is given as: reject H_o: $\ell^T \mu_r = \mathbf{0}$ in favor of H_a: $\ell^T \mu_r \neq \mathbf{0}$, iff

$$\frac{(\ell^T \boldsymbol{r})^2}{\ell^T \boldsymbol{\Sigma}_r \ell} \geq \chi^2_{q,\alpha}. \tag{22}$$

$100(1-\alpha)\%$ Bonferroni simultaneous confidence intervals for $\ell^T \mu_r$ are given by:

$$(\ell^T \boldsymbol{r} - c,\ \ell^T \boldsymbol{r} + c) \tag{23}$$

where

$$c = z_{\alpha/2k} \sqrt{\ell^T \boldsymbol{\Sigma}_r \ell} \tag{24}$$

and k equals the number of Bonferroni confidence intervals. Bonferroni intervals will be shorter than those given by Eq. 21 when c is less than b. In other words, Bonferroni intervals are smaller if $(z_{\alpha/2k})^2$ is less than $\chi^2_{q,\alpha}$. For $k \leq q$ confidence statements (or intervals), Bonferroni intervals are usually shorter than those given by Eq. 21 (Johnson and Wichern, 1982). However, when $k > q$, the intervals given by Eq. 21 may be shorter than the Bonferroni intervals. This situation will be true in particular when $k \gg q$. Note that the size of the intervals given by Eq. 21 are independent of the number of intervals. The test that corresponds to the Bonferroni confidence interval is given as: reject H_o: $\ell^T \mu_r = \mathbf{0}$ in favor of H_a: $\ell^T \mu_r \neq \mathbf{0}$, iff

$$\frac{|\ell^T \boldsymbol{r}|}{\sqrt{\ell^T \boldsymbol{\Sigma}_r \ell}} \geq z_{\alpha/2k}. \tag{25}$$

The power function with any ℓ_i can be obtained using the test statistic. For example, when using the Bonferroni statistic (Eq. 25), the power function is given by:

$$\beta_i = \mathcal{P}\left[\frac{|\ell_i^T \boldsymbol{r}|}{\sqrt{\ell_i^T \boldsymbol{\Sigma}_r \ell_i}} \geq z_{\alpha/2k} \,|\, \ell_i^T \mu_r \right] \tag{26}$$

which can be transformed into the more useful form given below (see Bickel and Doksum, 1976; Mah and Tamhane, 1982; Rollins, 1990):

$$\beta_i = \Phi\left(\frac{\ell_i^T \mu_r}{\sqrt{\ell_i^T \boldsymbol{\Sigma}_r \ell_i}} - z_{\alpha/2k} \right) + \Phi\left(-\frac{\ell_i^T \mu_r}{\sqrt{\ell_i^T \boldsymbol{\Sigma}_r \ell_i}} - z_{\alpha/2k} \right) \tag{27}$$

where

$$\Phi(y) = \int_{-\infty}^{y} \frac{e^{-u^2/2}}{\sqrt{2\pi}} du \tag{28}$$

is the standard normal distribution function. Thus, if $\ell^T{}_i \mu_r \neq \mathbf{0}$, the probability of concluding that $\ell^T{}_i \mu_r = 0$ is $1 - \beta_i$. The power function for the test statistic given by Eq. 22 is:

$$\beta_i = \mathcal{P}\left[\frac{(\ell_i^T \boldsymbol{r})^2}{\ell_i^T \boldsymbol{\Sigma}_r \ell_i} \geq \chi^2_{q,\alpha} \,|\, \ell_i^T \mu_r \right]$$

$$= \mathcal{P}[\text{noncentral } \chi^2_q \geq \chi^2_{q,\alpha} \,|\, \Delta_i^2] \tag{29}$$

where

$$\Delta_i^2 = \frac{(\ell_i^T \mu_r)^2}{\ell_i^T \boldsymbol{\Sigma}_r \ell_i}. \tag{30}$$

For given values of N, μ_r, α, q and k, the power of the statistics given by Eqs. 22 and 25 can be compared using Eqs. 27 and 29. When $N > 1$, $\boldsymbol{\Sigma}_r$ should be replaced by $N^{-1} \boldsymbol{\Sigma}_r$.

UBET Data Reconciliation

After unbiased estimates have been found using Eq. 15, unbiased estimates for process variables (that have known distributions and satisfy physical constraints) can be determined. Based on Eqs. 14 and 15, the estimate of θ_1 can be rewritten in terms of estimates for δs and γs as follows. Let

$$\hat{\theta}_1 = \begin{bmatrix} \hat{\delta}_1 \\ \hat{\gamma}_2 \end{bmatrix}, \tag{31}$$

$$\hat{\delta} = \begin{bmatrix} \hat{\delta}_1 \\ \mathbf{0} \end{bmatrix}, \tag{32}$$

and

$$\hat{\gamma} = \begin{bmatrix} \mathbf{0} \\ \hat{\gamma}_2 \end{bmatrix}. \tag{33}$$

Note that by partitioning B_1^{-1} as

$$B_1^{-1} = \begin{bmatrix} B^{11} & B^{12} \\ B^{21} & B^{22} \end{bmatrix}, \tag{34}$$

and r as

$$r = \begin{bmatrix} r_1^{u\times 1} \\ r_2^{u\times 1} \end{bmatrix}, \tag{35}$$

Equations 32 and 33 can be rewritten as

$$\hat{\delta} = \begin{bmatrix} \begin{bmatrix} \hat{\delta}_1 \\ \mathbf{0} \end{bmatrix} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} B^{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} r \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} B^{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} r_1 \\ \mathbf{0} \end{bmatrix} r_2 = B_{\hat{\delta}} y \tag{36}$$

and

$$\hat{\gamma} = \begin{bmatrix} \mathbf{0} \\ B^{21} r_1 + B^{22} r_2 \end{bmatrix} = B_{\hat{\gamma}} y, \tag{37}$$

where (see Mardia et al., 1979)

$$B^{11} = A_{11}^{-1}, \tag{38}$$

$$B^{12} = \mathbf{0}, \tag{39}$$

$$B^{21} = -M_{22}^{-1} A_{21} A_{11}^{-1}, \tag{40}$$

$$B^{22} = M_{22}^{-1}. \tag{41}$$

Using unbiased estimators for the θs, we are proposing the following estimator for μ:

$$\hat{\mu} = y - \hat{\delta} \tag{42}$$

which by using Eq. 36 can be written as

$$\hat{\mu} = y - B_{\hat{\delta}} y = Dy \tag{43}$$

with

$$\mathrm{Var}(\hat{\mu}) = D\Sigma D^T. \tag{44}$$

Equation 42 is an unbiased estimator for μ since

$$E[\hat{\mu}] = E[y - \hat{\delta}] = \mu + \delta - \delta = \mu. \tag{45}$$

Note that this estimator satisfies the constraint requirement since

$$A\hat{\mu} = Ay - A\hat{\delta} = M\hat{\gamma}. \tag{46}$$

In addition, the components of $\hat{\mu}$, $\hat{\mu}_i$, are distributed $N(\mu_i, e_i^T D\Sigma D^T e_i)$. [Exactly, $\hat{\mu}$ is distributed as a degenerate $N_{p-u}(\mu, D\Sigma D^T)$ random vector. Therefore, $e_i D\Sigma D e_i$ must be greater than zero to use Eq. 47 for μ_i.] $100(1-\alpha)\%$ simultaneous confidence intervals for μ_i $(i = 1, \ldots, p)$ are, therefore,

$$\hat{\mu}_i \pm z_{\alpha/2p} \sqrt{e_i^T D\Sigma D^T e_i}. \tag{47}$$

Thus, when θ_2 is $\mathbf{0}$, the estimators for the components of μ are unbiased (Eq. 45), satisfy the constraint equation (Eq. 46) and have known distributions, which are used to give confidence intervals for the components of μ (Eq. 47).

Comparative Study

The features of this approach are shown by identification and estimation results, which were generated using the process network in Figure 1. This recycle network was taken from Narasimhan and Mah (1987).

In addition to performance results of this approach, comparative results of the serial compensation strategy (SCS) (Narasimhan and Mah, 1987) will also be presented. The SCS was chosen because it is capable of identifying δs and γs, and because its performance should be similar to other popular techniques based on the measurement test (Mah and Tamhane, 1982; Narasimhan and Mah, 1987; and Rollins, 1990).

It is not the purpose of this study to compare identification strategies, since we are not proposing one in this article. Rather it is to describe limiting behaviors of the test statistics of the proposed technique and for comparing them with the SCS. To examine the limiting behavior of the proposed test statistics, however, a strategy is needed to serve as a vehicle. For this purpose, we have chosen to use nodal strategies reported by Mah et al. (1976) and Heenan and Serth (1986). The strategies that we used varied in the number and types of tests, depending

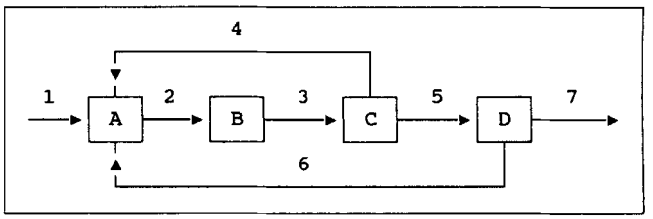

Figure 1. Recycle process network.

on the analysis (for example, whether one or two δs are considered). We varied them to keep the number of component hypothesis tests to a minimum. Since the Bonferroni test statistics were used, this procedure helped to maximize power and show limiting behavior with a minimum number of samples.

The study in this section is wider in scope than the one presented by Narasimhan and Mah (1987). In their study, only one nonzero θ existed and this knowledge was assumed known *a priori*. Thus, their identification procedure stopped after detecting one nonzero θ. For the SCS and this study, the number of nonzero θs was assumed to be unknown. Thus, the identification strategy stopped searching for nonzero θs only if it failed to find a nonzero θ: that is, this study incorporated the sequential SCS. In addition, the following variations, which were not included in the Narasimhan and Mah (1987) study, were included in this study: $N>1$, two nonzero θs, and $\Sigma \neq I$.

The performance measure that this study used was the *expected fraction of correct identification*. We called this value the overall performance (OPF). Note that, by definition, the OPF is bounded by 0 and 1. For a specific set of conditions (N, Σ, and so on), the OPF is fixed. For the cases using the Bonferroni test statistics and the nodal strategy (BNS), the OPFs were conservatively determined using laws of probability as shown in the Appendix.

The technique in the Appendix could not be used to determine OPFs for the SCS because the power function for this strategy is unknown. Thus, the SCS results were determined from simulated data. A digital DECstation 2100 workstation was used to generate the simulated data and obtain the OPF results. Each SCS result presented is based on 10,000 simulated trials. Thus, they should be very close to the expected fraction of correct identification, that is, the true OPFs.

The SCS OPFs were determined using the following equation:

$$\text{OPF} = \frac{\text{No. of trials with perfect identification}}{\text{No. of simulation trials (10,000)}}. \quad (48)$$

For example, a SCS OPF of 0.5 means that in the 10,000 simulated trials there were 5,000 trials that identified all zero θs correctly and all nonzero θs correctly. Note that Narasimhan and Mah (1987) used two performance measures in their study: the average type I error (AVTI) and the overall power (OP). They are defined, respectively, as:

$$\text{AVTI} = \frac{\text{No. of zero } \theta\text{s wrongly identified}}{\text{No. of simulation trials (10,000)}} \quad (49)$$

$$\text{OP} = \frac{\text{No. of nonzero } \theta\text{s correctly identified}}{\text{No. of nonzero } \theta\text{s simulated}} \quad (50)$$

This study presents the SCS results using all three measures of performance since it was easy to obtain the AVTI and OP values from the simulated cases. For comparison, however, we are interested only in high OPF values. Hence, we did not obtain AVTI and OP values for the BNS cases. Note that when an OPF is close to one, the AVTI will be small and the OP will also be close to one.

The performance results indicate that there are some common elements throughout all discussions. First, when $w=7$, only measurement biases for the seven streams are considered to be possible. When $w=9$, measurement bias on the seven streams and leaks at nodes B and C are considered possible.

SCS algorithm check

The SCS algorithm used in this study was developed from the information in Narasimhan and Mah (1987). First, results obtained by our SCS computer algorithm were compared with those from their SCS algorithm, as shown in Table 1. Runs 2.1c, 2.2c, and 2.3c in Table 1 are results from Narasimhan and Mah (1987) and the others are from our SCS algorithm. (As stated earlier, SCS results have a maximum possible detection of one.) Since the agreement is excellent as shown by AVTI and OP values, it was concluded that our SCS algorithm is correct.

A comparison of the Bonferroni and Sidak inequalities was made in Table 1 because Narasimhan and Mah (1987) used the Sidak inequality and we prefer to use the Bonferroni inequality. We prefer the Bonferroni inequality because unlike the Sidak inequality, it does not assume independence. By comparing the a runs with the b runs (or c runs) in Table 1, the values for $1-\beta^*$ (that is, Sidak) are shown to be only slightly smaller than values for $1-\alpha^*$ (that is, Bonferroni). As a result, the OPFs of b runs are only slightly better than those

Table 1. Comparison of Algorithm Results When $N=1$ and $\Sigma=I$

Run No.*	Algorithm Type**	$1-\beta^*$ or $1-\alpha^*$	w	Nonzero δ or γ	AVTI	OP	OPF
2.1a	R/D	0.9800	7	$\delta_1=5.0$	0.0615	0.8948	0.8948
2.1b	R/D	0.9787	7	$\delta_1=5.0$	**0.0608**	**0.8927**	0.8927
2.1c	N/M	0.9787	7	$\delta_1=5.0$	**0.06**	**0.90**	NR†
2.2a	R/D	0.9844	9	$\delta_1=5.0$	0.1385	0.8129	0.8129
2.2b	R/D	0.9834	9	$\delta_1=5.0$	**0.1396**	**0.8137**	0.8137
2.2c	N/M	0.9834	9	$\delta_1=5.0$	**0.14**	**0.82**	NR†
2.3a	R/D	0.9844	9	$\gamma_2=4.25$	0.1214	0.8691	0.8691
2.3b	R/D	0.9834	9	$\gamma_2=4.25$	**0.1214**	**0.8687**	0.8687
2.3c	N/M	0.9834	9	$\gamma_2=4.25$	**0.11**	**0.88**	NR†

* The Sidak inequality was used for the b and c runs. The Bonferroni inequality was used for the a runs. In all runs, $\alpha=0.14$ and $1-\beta^*=(1-\alpha)^{1/w}$ (the b and c runs) and $1-\alpha^*=1-\alpha/w$ (the a runs). Note the closeness of $1-\beta^*$ and $1-\alpha^*$.
** R/D are the results from the algorithm developed for this work. N/M are the results reported by Narasimhan and Mah (1987).
† NR means that this result was not given in Narasimhan and Mah (1987).

Table 2. SCS and BNS Results When $\delta_i = 5$, $\alpha = 0.05$, $w = 7$, and $\Sigma = I$

Run No.	i	N	AVTI SCS	OP SCS	OPF SCS	OPF BNS*
1	1	1	0.0679	0.8543	0.8368	0.2712
2	1	3	0.0268	0.9983	**0.9731**	**0.9301**
3	2	1	0.0680	0.8583	0.8427	0.2712
4	2	3	0.0277	0.9985	**0.9727**	**0.9301**
5	3	1	0.0672	0.8517	0.8354	0.4235
6	3	3	0.0258	0.9984	**0.9745**	**0.9621**
7	4	1	0.0923	0.7201	0.7127	0.0619
8	4	3	0.0204	0.9956	**0.9792**	**0.9226**
9	5	1	0.0763	0.8135	0.8008	0.2142
10	5	3	0.0219	0.9977	**0.9787**	**0.9546**
11	6	1	0.0915	0.7247	0.7173	0.0619
12	6	3	0.0206	0.9966	**0.9795**	**0.9226**
13	7	1	0.0705	0.8548	0.8372	0.4235
14	7	3	0.0274	0.9984	**0.9729**	**0.9621**

* For BNS, $k = 5$: all four nodal balances and the overall mass balance are used only.

of a runs. Since Sidak's appeared to have no significant advantage over Bonferroni's, the remainder of the work was done using the Bonferroni inequality. Rollins (1990) offers additional comments and comparisons of these two inequalities.

Equal and Unit Measurement Variances ($\Sigma = I$)

The study of Narasimhan and Mah (1987) for this process was done with $\Sigma = I$ in all cases. We, however, examine cases with $\Sigma = I$ and with $\Sigma \neq I$, since Σ is not likely to equal I in practice. The $\Sigma = I$ results are in Tables 2 to 5.

Table 2 shows cases with $w = 7$, $N = 1$ and 3, and $\delta_i = 5$ (when $\delta_j = 0$ for $i \neq j$). OPF is above 0.9 (for $N = 3$) for both SCS and BNS in all cases. OPF appears to be better for SCS than BNS for $N = 1$, but the difference rapidly disappears as N increases. Note that the BNS results will be conservative since they were determined using a Bonferroni inequality procedure: the actual OPF will be greater than those reported in this work. However, since we are mainly interested in whether or not we can obtain excellent performance (OPF above 0.9), this conservative approach is sufficient for this study.

For the same conditions as those in Table 2, except for $w = 9$, similar results were obtained for all measured streams. These results are not shown to conserve space. However, the SCS result, as shown in Table 3 for cases when $\gamma_B = 5$ and for cases when $\gamma_C = 5$, is 0.0. As Table 3 shows, the low OPFs are due to very high AVTIs. In another study (not tabulated here), N was increased incrementally from 1 to 5 to study its effect on

Table 3. SCS and BNS Results When $\theta = 5$, $\alpha = 0.05$, $w = 9$, and $\Sigma = I$

Run No.	θ	N	AVTI SCS	OP SCS	OPF SCS	OPF BNS*
1	γ_B	1	2.7186	0.9582	**0.0000**	0.6328
2	γ_B	3	3.3549	1.0000	**0.0000**	0.9696
3	γ_C	1	2.5831	0.8986	**0.0000**	0.4235
4	γ_C	3	3.1830	0.9974	**0.0000**	0.9621

* For the BNS, $k = 5$: all four nodal balances and the overall mass balance are used only.

Table 4. SCS and BNS Results When $\delta_1 \neq 0$ and $\delta_2 \neq 0$, $\alpha = 0.05$, $w = 7$, and $\Sigma = I$

Run No.	δ_1	δ_2	N	AVTI SCS	OP SCS	OPF SCS	OPF BNS
1	5.0	5.0	1	0.1820	0.7987	0.6801	**0.050**
2	5.0	5.0	2	0.0868	0.9669	0.9387	**0.050**
3	5.0	5.0	3	0.0375	0.9909	0.9768	**0.050**

SCS performance. As N increased, both OP and AVTI increased. At $N = 5$, OP = 0.9999, but AVTI = 3.8104. Thus, it does not appear that SCS is capable of accurate performance (that is, high OPFs) for leaks at nodes B or C when $w = 9$.

Cases of "true" error cancellation in Table 4 are for $\delta_1 = \delta_2 = 5$. The results show that the SCS was able to achieve excellent OPF performance, but the BNS was not able to. The BNS OPF is simply at the α level of 0.05 (because the test statistics have an expected value of zero at all levels of N). Thus, the SCS appears to have a significant power advantage over the BNS when θs have a canceling effect. However, note that unless there is perfect cancellation (which is not likely), the BNS will be capable of achieving large values of OPF. In another study, not tabulated here, when δ_1 and δ_2 are nonzero and have an adding effect, we found that the performance of both approaches is essentially equal for all values of N from 1 to 5.

Table 5 compares all combinations of two δs simultaneously not equal to zero ($w = 7$). As indicated, BNS was unable to reach numerical conclusions on nine cases. This happened because these δs are distributed such that their unique estimation is not possible. That is, A_{11} is not of rank 3 when θ_1 have the components shown in the last column of Table 5 (1,6,7; 2,3,4; and 4,5,6). It is important to note, however, that in all these

Table 5. SCS and BNS Results When $\delta_i = 7$, $\delta_j = 4$, $\Sigma = I$, $\alpha = 0.05$, $w = 7$, and $N = 10$

Run No.	i	j	AVTI SCS	OP SCS	OPF SCS	OPF BNS*
1	1	2	0.0138	1.0000	0.9862	0.9588
2	1	3	0.0116	1.0000	0.9884	0.9749
3	1	4	0.0188	1.0000	0.9812	0.9900
4	1	5	0.0827	0.9953	0.9226	0.9881
5	1	6	0.8733	0.9998	**0.1550**	**1,6,7****
6	1	7	1.0924	0.5685	**0.0000**	**1,6,7****
7	2	3	1.0908	0.5665	**0.0000**	**2,3,4****
8	2	4	0.9131	0.5979	**0.1071**	**2,3,4****
9	2	5	0.0836	0.9944	0.9276	0.9796
10	2	6	0.0204	0.9999	0.9799	0.9658
11	2	7	0.0135	1.0000	0.9865	0.9830
12	3	4	0.9137	0.5972	**0.1074**	**2,3,4****
13	3	5	0.0816	0.9943	0.9294	0.9866
14	3	6	0.0190	1.0000	0.9813	0.9567
15	3	7	0.0128	1.0000	0.9872	0.9679
16	4	5	1.1041	0.7207	**0.0000**	**4,5,6****
17	4	6	0.5426	1.0000	**0.0000**	**4,5,6****
18	4	7	0.0527	1.0000	0.9473	0.9900
19	5	6	1.0217	0.6152	**0.0249**	**4,5,6****
20	5	7	0.0960	0.9998	0.9046	0.9726
21	6	7	0.9198	1.0000	**0.1063**	**1,6,7****

* For BNS, only the following balances were considered: A, B, C, D, ABCD, ABC, BC, CD, and AB.
** At least two of the three measurements are biased.

Table 6. SCS and BNS Results When $\delta_i = 10.0$, $\alpha = 0.05$, $w = 7$, $N = 5$, and $\Sigma \neq I$

Run No.	i	AVTI SCS	OP SCS	OPF SCS	OPF BNS**
1	1	0.0305	1.0000	0.9695	0.9558
2	2	0.0347	0.9966	0.9655	0.9481
3	3	0.0320	0.99667	0.9682	0.9623
4	4	0.0218	0.9999	0.9783	0.9558
5	5	0.0201	1.0000	0.9801	0.9700
6	6	0.2698	0.8752	**0.8574**	0.9558
7	7	0.3088	1.0000	**0.6939**	0.9700
8*	7	0.9972	1.0000	**0.0831**	0.9700

* $N = 10$.
** $k = 5$: all four nodal balances and the overall mass balance are used.

cases BNS reaches the conclusion that at least two of three δs are nonzero, which is correct. For all the cases, in which BNS was not able to reach numerical conclusions, SCS performed very poorly. In all these runs, the AVTIs are poor and OPs are poor in about two-thirds of them. Thus, SCS does not appear to have an advantage over BNS in these cases. Furthermore, we argue that BNS offers an advantage because of the correct conclusions that it is able to reach and because it gives a direct indication when estimation is not possible. SCS cases were also run (not shown) with $\delta_2 = 7.0$, $\delta_3 = 3.0$, and N varying from 1 to 10. In these cases, as N increased, the OPF did not improve. Thus, a large N is not likely to provide acceptable SCS performance for these conditions.

Unequal Measurement Variance ($\Sigma \neq I$)

Tables 6 and 7 examine similar cases for conditions when $\Sigma = \text{diag}\ (10, 10, 10, 1, 1, 1, 1)$. Table 6 shows the cases with one nonzero δ, and Table 7 the cases with two nonzero δs. Table 6 shows that the SCS OPFs for Runs 6 ($\delta_6 = 10.0$) and 7 ($\delta_7 = 10.0$) are below 0.9. For run 8, $\delta_7 = 10.0$ and $N = 10$. As this run indicates, the SCS OPF dropped dramatically due to a substantial increase in the AVTI. Thus, the ability of SCS to achieve OPFs, which are greater than 0.9, depends on the value of Σ. In contrast, the value of Σ did not affect the ability of BNS to achieve OPFs, which are greater than 0.9, as supported by Table 6. In general, the value of Σ does not affect the ability of BNS to obtain high OPFs.

All the BNS cases that were shown to be capable of obtaining excellent performance in Table 5 have excellent OPF performances also in Table 7. The same cannot be said of SCS, however. Note that only three out of 21 SCS cases have OPFs > 0.9. The value of Σ is shown to have a very significant effect on SCS performance, especially when more than one δ is nonzero.

Estimation performance

Table 8 lists estimation results for both approaches, but only the cases from Table 5 with superior performance. The results for SCS were determined from the simulated cases with perfect identification. Since each SCS result is based on more than 9,000 values, the values for the means of the estimates (δ_k) and the standard deviations of the estimates ($\sigma_{\delta k}$) should be close to their true values. The BNS results were determined directly from the true means and standard deviations.

Table 8 clearly shows that even when identification performance is perfect, the SCS estimates of δ can be significantly biased. It also shows that some of the SCS standard deviations of the estimates are twice as large as the standard deviations for some of the corresponding BNS values. Note, however, that there are some cases where SCS standard deviations are slightly (but still significantly) smaller than the BNS standard deviations. These results show that on the average the SCS estimates can have much larger errors than the BNS estimates. Rollins (1990) discussed how the SCS estimators can be even more biased (that is, inaccurate) as the number of nonzero θs increase.

Table 7. SCS and BNS Results When $\delta_i = 15$, $\delta_j = 7.5$, $\alpha = 0.05$, $w = 7$, $N = 10$, and $\Sigma \neq I$

Run No.	i	j	AVTI SCS	OP SCS	OPF SCS	OPF BNS*
1	1	2	0.0181	0.9993	0.9820	0.9662
2	1	3	0.0167	0.9995	0.9833	0.9732
3	1	4	0.0094	1.0000	0.9906	0.9960
4	1	5	2.7108	0.5150	**0.0000**	0.9789
5	1	6	0.6613	0.7163	**0.3479**	**1,6,7****
6	1	7	1.0202	0.5000	**0.0000**	**1,6,7****
7	2	3	0.8459	0.9128	**0.1761**	**2,3,4****
8	2	4	1.0227	0.5000	**0.0000**	**2,3,4****
9	2	5	2.1069	0.5005	**0.0009**	0.9719
10	2	6	1.9999	0.5015	**0.0028**	0.9567
11	2	7	1.3084	0.7852	**0.5510**	0.9830
12	3	4	1.0218	0.5000	**0.0000**	**2,3,4****
13	3	5	2.1075	0.5035	**0.0060**	0.9889
14	3	6	1.9991	0.5017	**0.0031**	0.9567
15	3	7	1.3030	0.7860	**0.5533**	0.9830
16	4	5	1.4509	0.5201	**0.0000**	**4,5,6****
17	4	6	0.6101	0.9382	**0.5620**	**4,5,6****
18	4	7	0.8369	1.0000	**0.3142**	0.9900
19	5	6	1.3916	0.7832	**0.0000**	**4,5,6****
20	5	7	2.5551	1.0000	**0.0000**	0.9830
21	6	7	1.1506	0.6590	**0.0000**	**1,6,7****

* Only the following balances were considered: A, B, C, D, ABCD, ABC, BC, CD, and AB.
** At least two of the three measurements are biased.

Concluding Remarks

New statistical equations for gross error detection are presented for chemical process operations. They include:

- Power function for the global test
- Two identification test statistics and their power functions
- Estimates and confidence intervals for measurement biases and leaks
- Estimates and confidence intervals for measured process variables.

Overall the results show that this unbiased estimation approach has merit in the area of gross error detection and should provide the basis for improvements in identification and reconciliation strategies.

The statistical objectives of this approach center on highly accurate identification and obtaining unbiased estimates. The proposed test statistics have the potential to contribute significantly to the identification objective, even though it will require the development of an effective strategy for their implementation. This article has shown how to achieve high identification accuracy and how it is possible to obtain unbiased

Table 8. SCS and BNS Estimation Results When $\alpha = 0.05$, $N = 10$, $\delta_i = 7.0$, $\delta_j = 4.0$ and $\Sigma = I$

			SCS			BNS	
i	j	k	N_k*	δ_k	$\sigma_{\delta k}$	δ_k	$\sigma_{\delta k}$
1	2	1	9862	6.69	0.5089	7.0	0.6325
		2	9862	3.98	0.3996	4.0	0.4472
1	3	1	9884	6.69	0.5095	7.0	0.6325
		3	9884	3.98	0.4045	4.0	0.4472
1	4	1	9812	**7.61****	0.7340	7.0	0.7071
		4	9812	3.87	0.4720	4.0	0.5477
1	5	1	9266	**6.08****	**1.0092****	7.0	0.6325
		5	9266	3.76	0.4726	4.0	0.5477
2	5	2	9276	**6.08****	**1.0062****	7.0	0.4472
		5	9276	3.76	0.4736	4.0	0.5477
2	6	2	9799	**6.38****	0.7338	7.0	0.4472
		6	9799	3.87	0.4741	4.0	0.5477
2	7	2	9865	6.69	0.5046	7.0	0.4472
		7	9865	3.98	0.4031	4.0	0.5477
3	5	3	9294	**6.08****	**1.0037****	7.0	0.4472
		5	9294	3.76	0.4714	4.0	0.5477
3	6	3	9813	**6.39****	0.7346	7.0	0.5477
		6	9813	3.87	0.4789	4.0	0.5477
3	7	3	9872	6.69	0.5073	7.0	0.5477
		7	9872	3.98	0.4029	4.0	0.5477
4	7	4	9473	**7.80****	**0.9210****	7.0	0.4472
		7	9473	3.87	0.4167	4.0	0.5477
5	7	5	9046	**6.00****	**1.0830****	7.0	0.4472

* No. of SCS simulation trials with perfect identification.
** Most biased estimates and largest standard deviations.

estimators. Without restrictions on sample size, the proposed test statistics with a nodal strategy are capable of superior performance to the SCS in all but one situation examined—"true" error cancellation: in other words, they showed superiority in identifying leaks and in cases where $\Sigma \neq I$. Also, the proposed estimators showed overall superiority in accuracy.

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Notation

$\boldsymbol{A}$ = process constraint matrix
$\boldsymbol{A}_{ij}$ = ith row, jth column partition of $\boldsymbol{A}^*$
AVTI = average type I error
$\boldsymbol{B}_i$ = ith partition of $\boldsymbol{B}$
$\boldsymbol{B}^{ij}$ = ith row, jth column of $\boldsymbol{B}_1^{-1}$
$\boldsymbol{B}_{\tilde{\delta}}$ = matrix given by Eq. 36
$\boldsymbol{B}_{\tilde{\gamma}}$ = matrix given by Eq. 37
BNS = Bonferroni nodal strategy
b = constant given by Eq. 20
c = constant given by Eq. 24
$\boldsymbol{D}$ = matrix given by Eq. 43
$E[x]$ = expected value of x
$\boldsymbol{e}_i$ = vector with 1 in the ith place and 0 elsewhere
GED = gross error detection
GT = global test
H_a = alternative hypothesis
H_o = null hypothesis
iff = if and only if
k = number of tests or confidence intervals
ℓ = vector used for making linear combinations of measurements
$\boldsymbol{M}$ = diagonal matrix of constants for calculating leaks
$\boldsymbol{m}_j$ = a vector with a value in the jth place and zeros elsewhere
m_j = a constant for leaks in node j
N_q = multivariate normal q distribution
N_p = multivariate normal p distribution
N = number of samples
OP = overall power
OPF = overall performance
$\mathcal{P}$ = probability
$\mathcal{P}(A|B)$ = probability of A occurring given that B has occurred
p = number of measurements or variables
q = number of constraints
$\boldsymbol{r}$ = transformed measurement vector $(\boldsymbol{Ay})$
r_i = ith element of $\boldsymbol{r}$
SC = serial compensation
SCS = serial compensation strategy
UBET = unbiased estimation technique
w = number of possible nonzero θs
$\boldsymbol{y}$ = vector of measurements
$z_{\alpha/2q}$ = $100(\alpha/2q)$th percentile of the normal distribution

Greek letters

α = type I error level
β = power, 1 − type II error level
β_i = power for the ith test
$\boldsymbol{\delta}$ = unknown vector of measurement biases
$\boldsymbol{\delta}_i$ = ith partitioning of $\boldsymbol{\delta}^*$
Δ^2 = noncentrality parameter
δ_i = ith element of $\boldsymbol{\delta}$
$\boldsymbol{\epsilon}$ = vector of random errors
$\boldsymbol{\gamma}$ = unknown process leak vector
$\Phi(\cdot)$ = normal distribution function

μ = unknown vector of true means
$\hat{\mu}$ = estimate of μ
μ_r = $E[r]$
μ_y = $E[\mu_y]$
$\hat{\mu}_y$ = estimate of μ_y
Σ = variance-covariance measurement matrix
Σ_r = variance-covariance matrix for r
σ_{ii} = ith diagonal element of Σ
σ_{rjj} = jth diagonal element of Σ_r
σ_{dii} = ith diagonal element of Σ_d
θ = vector with elements of δ and γ
θ_i = ith partition of θ
θ = ith element of θ
$\chi^2_{q,\alpha}$ = upper (100α)th percentile of the χ^2_q distribution
$\chi_{q,\Delta}$ = noncentral χ^2_q variable with noncentrality parameter Δ^2

Other symbols

$\sim$ = is distributed

Superscript

T = transpose

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Appendix: Example Calculation of OPF for BNS Cases

The following is an example of how the overall performance (OPF) is obtained for the BNS cases reported in Tables 2 to 8. The conditions for this example are $\delta_1 \neq 0$ and $\delta_j = 0$ for $j \neq 1$, and hypothesis tests will be performed only on the four nodal balances (A, B, C, and D) of Figure 1. Thus, $\mu_{rA} = \delta_1$, $\mu_{rB} = \mu_{rC} = \mu_{rD} = 0$. Let $A_1 = \{H_{o1}$: $\mu_{r1} = 0$, is rejected$\}$, $A_2 = \{H_{o2}$: $\mu_{r2} = 0$, is not rejected$\}$, $A_3 = \{H_{o3}$: $\mu_{r3} = 0$, is not rejected$\}$, and $A_4 = \{H_{o4}$: $\mu_{r4} = 0$, is not rejected$\}$. Additionally, let $\mathcal{P}\{A_1\} = \beta$, $\mathcal{P}\{A_2\} = 1 - \alpha/k$, $\mathcal{P}\{A_3\} = 1 - \alpha/k$, and $\mathcal{P}\{A_4\} = 1 - \alpha/k$, where $k = 4$. Therefore, $\mathcal{P}\{\bar{A}_1\} = 1 - \beta$, $\mathcal{P}\{\bar{A}_2\} = 1 - (1 - \alpha/k) = \alpha/k$, $\mathcal{P}\{\bar{A}_3\} = \alpha/k$, and $\mathcal{P}\{\bar{A}_4\} = \alpha/k$. Hence,

$$\mathcal{P}\{A_1 \text{ and } A_2 \text{ and } A_3 \text{ and } A_4\}$$

$$= \mathcal{P}\{\text{all conclusions are correct}\} \quad \text{(A1a)}$$

$$= 1 - \mathcal{P}\{\text{at least one conclusion is incorrect}\} \quad \text{(A1b)}$$

$$\geq 1 - \mathcal{P}\{\bar{A}_1\} - \mathcal{P}\{\bar{A}_2\} - \mathcal{P}\{\bar{A}_3\} - \mathcal{P}\{\bar{A}_4\} \quad \text{(A1c)}$$

$$= \beta - \alpha/k - \alpha/k - \alpha/k \quad \text{(A1d)}$$

$$= \text{OPF} \quad \text{(A1e)}$$

Equation A1c is determined by the application of Bonferroni's method (see Johnson and Wichern, 1982, for example). Additionally, note that β is determined from Eq. 25.

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